The "Aha!" moment for α

For a given significance level, how do we build test?

Question

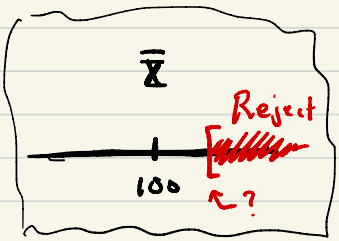
$X \sim N(\mu, 10)$

$n = 50$

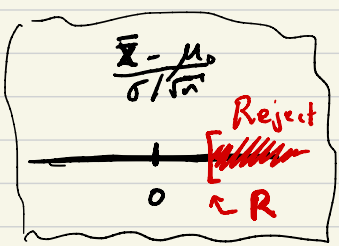
Suppose that we have a population that is normally distributed with unknown mean and standard dev 10.

Suppose we have 50 samples.

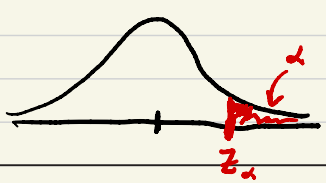
Construct a statistical test with $H_0: \mu = 100$, $H_1: \mu > 100$ so that its significance level is 0.2



Solution: test statistic: $\dfrac{\bar{X} - 100}{10/\sqrt{50}}$



Rejection Region: $R \longleftarrow z_\alpha$



$$\alpha = P(\text{Reject when } H_0 \text{ is true})$$

$$= P\left(\frac{\bar{X} - 100}{10/\sqrt{50}} \geq R\right)$$

So we want to find R so that $P(Z \geq R) = \alpha$.

This means we need to pick $R = z_\alpha$

The "Aha!" moment for α

Try this on your own

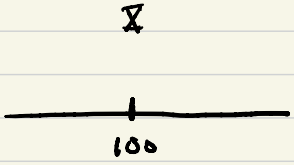
Question

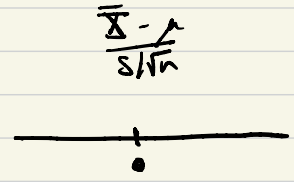
Two-sided test

Suppose that we have a population that is normally distributed with unknown mean and unknown st dev

Suppose we have 50 samples.

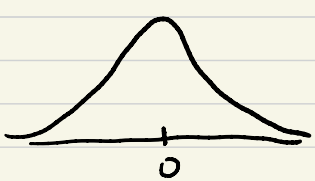
Construct a statistical test (Two-sided) with $H_0: \mu = 100$, $H_1: \mu \neq 100$ so that its significance level is 0.2





Solution Test statistic: $\dfrac{\bar{X} - 100}{S/\sqrt{n}}$

Rejection Region: _____



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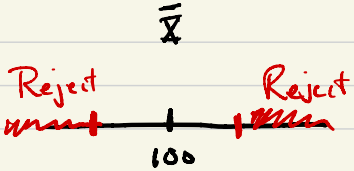
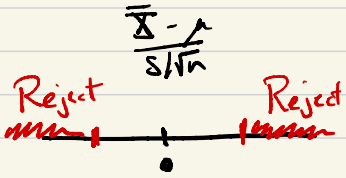
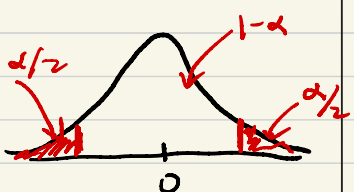




Solution Test statistic: $\dfrac{\bar{X} - 100}{S/\sqrt{n}}$

Rejection Region: $(-\infty, -t_{0.1}] \cup [t_{0.1}, \infty)$



Since $\alpha = P\left(-t_{0.1} \leq \dfrac{\bar{X} - 100}{S/\sqrt{n}} \leq t_{0.1}\right)$