## Math 1230: Stat for Scientists

Calculus: derivatives and integrals

**Problem 1.** Evaluate the following derivatives. a. Let  $f(x) = \frac{1}{2}x^3 + 3x^2 + e^{4x}$ . What is f'(x)?

b. Let  $g(y) = 3y - 4 + e^{-3y^2}$ . What is g'(y)? c. Let  $h(t) = \sqrt{3t} + \frac{8}{5t^2}$ . What is  $\frac{d}{dt}h(t)$ ?

**Problem 2.** Determine the antiderivative of each of the following functions. (Assume x > 0 in all cases.)

a.  $f(x) = 3x^2 - 4\sqrt{x}$ . b.  $g(x) = 3e^{-5x}$ . c.  $h(x) = x^3 + \frac{1}{x^3} - \frac{4}{x}$ .

Problem 3. Compute each of the following definite integrals.

a. 
$$\int_{0}^{4} x - 3x^{2} dx$$
.  
b.  $\int_{1}^{4} \sqrt{x} dx$ .  
c.  $\int_{2}^{5} e^{-2x} dx$ .

**Problem 4.** Find the value of *C* so that

$$\int_0^2 Cx^2 \mathrm{d}x = 1.$$

Problem 5. Consider the following piecewise-defined function:

$$f(x) = \begin{cases} 2x, & x \in [0,1] \\ 4 - 2x, & x \in (1,2] \\ 0, & \text{otherwise} \end{cases}$$

- a. Sketch a graph of this function.
- b. Is this a continuous function?
- c. For what values of *x* is this function differentiable?
- d. Write f'(x) as a piecewise-defined function.
- e. For all x > 0, determine  $F(x) = \int_0^x f(y) dy$ .
- f. Sketch F(x).