Math 1230, Fall 2022, Problems for Practice, Exam 1 *Exam: Sept 23, 2022.*

These problems are purely for practice and to help you test your understanding of material related to Exam 1. This should not be considered comprehensive in that there are topics that are eligible to appear on the exam that may not appear here.

In what follows, recall that the cumulative distribution function for a random variable X is always defined to be

$$F(x) = P(X \leq x).$$

1. Among 100 students, a survey shows that 75 like M&Ms and 60 like Skittles, while 15 people like neither.

   Let M be the event that a student likes M&Ms and S be the event that a student likes Skittles.

   (a) Express the event "a student likes M&Ms but does not like Skittles" as a mathematical phrase.

   (b) Express the event $M \cap S$ as a common language phrase.

   (c) Compute $P(M \cap S)$. (Hint: first determine $P(M \cup S)$ from the information given.)

2. Define $I = \{x \,|\, 0 < x \leq 10\}$ and define $J = [-2, 2] \cup (5, 10)$

   Find $I \cap J'$. (You may express your answer using interval or set (rule) notation.)

   (Prof comment: This problem is a little trickier than what I would put on the exam, but it is good practice to pay attention to the details in this one.)

3. Roughly 5% of men are colorblind, while 0.25 percent of women are colorblind. Assuming the population consists of 50% men and 50% women, if a random colorblind person is selected, what it the probability that person is male?

4. There are 15 students who need to get downtown and only 3 cars available to get them there. One car holds 6 people, while another holds 5, and a third holds 4.

   (a) How many different ways can the students arrange themselves to get down town?

   (b) (Challenge question: tougher than an exam question, but worth thinking about) What if four of the students declared that they are BFF's and they all have to be in the same car together?

5. (a) How many ways are there to rearrange the letters in the word WORD?

   (b) How many ways are there to rearrange the letters in the word ARRANGE?

6. In a certain college, 52% of the students are female. Of all students, 5% are majoring in Philosophy. Of all students, 3% are both female and majoring in Philosophy.

   (a) What is the probability that a student is female, given that the student is a Philosophy major?

   (b) What is the probability that a student is a Philosophy major, given that the student is female?

7. Suppose that an individual is randomly selected from the population of all adult males living in the United States. Let A be the event that the selected individual is over 6 ft in height, and let B be the event that the selected individual is a professional basketball player. Which do you think is larger, $P(A|B)$ or $P(B|A)$, and why?

8. Suppose that a random variable X has probability density function

$$f(x) = \begin{cases} (x+1), & -1 \le x \le 0 \\ \frac{1}{4}(2-x), & 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

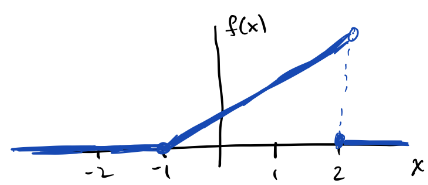
Find the mean of this distribution. Draw a (careful) sketch of this density and show that your answer makes sense.

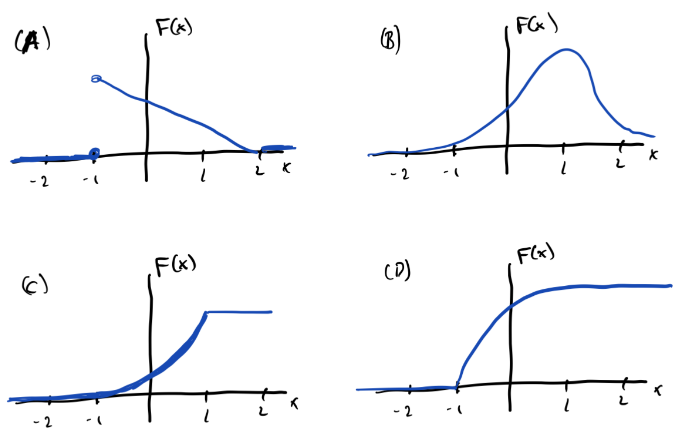
9. Suppose that a random variable X has a probability density function

$$f(x) = \begin{cases} \dfrac{C}{2+x}, & 0 \le x \le 3 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the value of C.

10. Consider the following density function $f(x)$. Which of the following functions could be the corresponding cumulative distribution function $F(x)$?





11. Suppose that X is a **discrete** random variable the has the following values for its probability mass function and cumulative distribution function.

X	0	1	2	3	4	5
p.m.f.	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	???	0	$\frac{1}{6}$
c.d.f.	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	???	???	???

(a) Fill in the missing values.

(b) Compute the mean of this distribution.

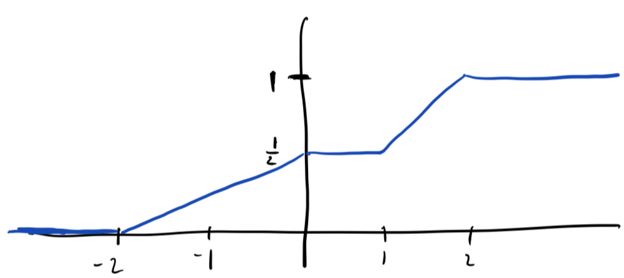
(c) Compute $P(X \in [1, 3])$.

12. Suppose that X is a **continuous** random variable the has the following values for its probability density function and cumulative distribution function.

X	0	1	2	3	4	5
p.d.f.	0	0.08	0.16	0.24	0.32	0.40
c.d.f.	0	0.04	0.16	0.36	0.64	1

Compute $P(X \in [1, 3])$.

13. Consider a **continuous** random variable X that has the cumulative distribution function depicted here.

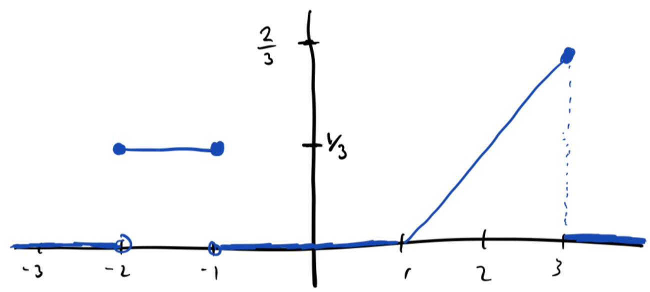


(a) What is $P(X \leq 3)$?

(b) What is $P(0 < X \leq 1)$?

(c) What is $P(-2 \leq X \leq 2)$?

14. Consider a **continuous** random variable X that has the probability density function depicted here.



(a) What is $P(X \leq 3)$?

(b) What is $P(0 < X \leq 1)$?

(c) What is $P(-2 \leq X \leq 2)$?

15. Suppose that a **discrete** random variable X has the following probability mass function:

$$f(0) = \frac{1}{6}, \quad f(1) = \frac{1}{2}, \quad f(2) = \frac{1}{3}, \quad \big(\text{with } f(x) = 0 \text{ otherwise.}\big)$$

(a) Compute $E(X^3)$.

(b) Compute $P(X > 1.5)$.

(c) Compute $F(1)$, where F is the cdf of X.