Math 1230, Fall 2022, Problems for Practice, Exam 2

These problems are purely for practice and to help you test your understanding of material related to Exam 2. This should not be considered comprehensive in that there are topics that are eligible to appear on the exam that may not appear here. Your midterm will feature several multiple choice questions.

1. Suppose that random variables X and Y are independent and have the following properties:

$$E(X) = 5,\ \mathrm{Var}(X) = 2,\ E(Y) = -2,\ E(Y^2) = 7.$$

   Compute the following.

   (a) $E(X + Y)$.

   (b) $\mathrm{Var}(2X - 3Y)$

   (c) $E(X^2 + 5)$

   (d) The standard deviation of Y.

2. Consider the following data set:

$$\vec{x} = \{90, 88, 93, 87, 85, 95, 92\}$$

   (a) Compute $\bar{x}$.

   (b) Compute the standard deviation of this set.

3. Suppose that there is a classroom with 20 students. Each of those students independently flips a fair coin.

   (a) What is the probability that exactly 13 students will flip HEADs and 7 students will flip TAILs? (Do not use a table)

   (b) Use the binomial table for this question. What is the probability that at least 13 students will flip HEADs?

   (c) Is the outcome of 13 HEADs more than one standard deviation above expected outcome?

4. Suppose that there is a classroom of 200 students. Among those students, 50 are from the West, 75 are from the East, 25 are from the North, and 50 are from the South. Suppose I decide to form a committee of 8 students.

   (a) What is the probability that there will be exactly 3 students from the South?

   (b) What is the expected number of students from the South?

   (c) What is the standard deviation for the number of students from the South?

5. Suppose that Z is a standard normal random variable.

   (a) Compute $P(Z > 1.5)$.

   (b) What is the probability that Z is larger than 1.5 given that it is positive?

   (c) Find a value k such that $P(-k \leq Z \leq k) \approx 0.2)$.

6. Suppose that in a city of 10,000 people, there are 4,000 who like football and 6,000 who do not.

   (a) Suppose we conduct a poll of 16 citizens. Let X be number of those polled who like football. What is the probability that $X < 8$? (Use Binomial approximation to find a decimal answer.)

   (b) Suppose we conduct a poll of 64 citizens. Let Y be number of those polled who like football. What is the probability that $Y < 32$? (Use Normal approximation and be sure to include the continuity correction.)

7. Suppose that $X_1, X_2, \ldots X_{10}$ are independent and identically distributed continuous random variables that each have mean 10 and a variance 4. Let $\bar{X} = \dfrac{1}{10}\displaystyle\sum_{i=1}^{10} X_i$.

   (a) What are $E(X_1)$ and $E(\bar{X})$?

   (b) What are $\mathrm{SD}(X_1)$ and $\mathrm{SD}(\bar{X})$?

   (c) Use normal approximation of X_1 to give an approximate value for $P(X_1 > 12)$.

   (d) Use the CLT to give an approximate value for $P(\bar{X} > 12)$.

8. Suppose that X is an Exponential random variables with parameter $\beta = 3$.

   (a) What is $E(X)$?

   (b) Compute $P(X > 2)$.

   (c) Compute $P(X > 5 \,|\, X > 3)$.

   (d) Suppose that $X_1, X_2, \ldots X_{25}$ are independent Exponential random variables and let $\bar{X} = \frac{1}{25}\sum_{i=1}^{25} X_i$. Use the CLT to approximate $P(\bar{X} > 2)$.

9. An assembly line at a plant produces exactly 10000 widgets a day. Suppose that approximately 1 out of every 2000 fails a standards test and is thrown out. What is the probability that there will be 10 or more widgets thrown out on a given day?

10. There are 100 green balloons and 150 red balloons in a bag. Suppose we extract 10 balloons from the bag.

    (a) What is the exact probability that five of the balloons will be green?

    (b) Use Binomial Approximation to find the probability that exactly five of the balloons will be green.

    (c) Use Binomial Approximation to find the probability that no more than four of the balloons are green.

11. Suppose that X is a continuous random variable and is uniformly distributed over the interval $[10, 20]$.

    (a) What is $E(X)$ and $\mathrm{SD}(X)$?

    (b) What is $P(X > 13)$?

    (c) What is $P(X > 13 \,|\, X < 17)$?

12. The heights of women in the US are roughly normally distributed with a mean of 54 inches and a standard deviation of 3 inches.

    (a) Suppose there will be a new student attending class next week. What is the probability that she will be taller than 60 inches?

    (b) Let X represent the height of a new student. For what value k is it true that $P(X > k) = 0.01$?

13. Historically, the grade distribution for a certain test has been normally distributed with a mean of 80 and a standard deviation of 5. Sally is taking the exam next week and wants to know the probability that she will score at least a 90. If she assumes her outcome will be like those of students who have taken the class in the past, what is this probability?

14. Suppose that a business owner is studying whether to open at 7:00 or 8:00 am. For a month (30 days), each morning she records the number of customers that arrive during that hour. For the following questions, assume that the true (unknown to her) arrival rate of customers between 7:00 and 8:00 am is 5 per hour.

    (a) What is the expected arrival time of the first customer?

    (b) What is the probability that the first customer will arrive after 7:30?

    (c) What is the probability that zero customers will arrive between 7:00 and 8:00 am?

    (d) Among the 30 days that she made observations, what is the expected number of days in which zero customers will arrive in the first hour?

    (e) Let $\bar{X}$ be the average number of arrivals over a 30 day sample. Use the CLT to approximate $P(4 \leq \bar{X} \leq 6)$. *You may ignore the continuity correction.*

15. Suppose that the number of bees in a flower patch is roughly 5 per square meter. A gardener has a flower patch that is 3 m by 1 m.

    (a) What is the probability that there will be between 10 and 20 bees in the patch (inclusive of 10 and 20)?

    (b) What is the largest value k such that $P(X \leq k) = 0.05$?