Math 1230: Stat for Scientists **Practice Problems, Exam 3**

Major topics: The t and chi-squared distributions, Point Estimation, Confidence Intervals, Hypothesis Testing, Signficance, Power, and p-values

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Problem 1. Suppose that T is t-distributed with 17 degrees of freedom.

(a) What is $P(T > 2.110)$?

(b) What is $P(T < 0.534)$?

(c) Find a value k so that $P(T > k) = 0.01$.

(d) Estimate $P(T > 3)$.

Problem 2. Determine the following critical values. For the t and chi-squared distributions, assume the number of degrees of freedom is $v = 10$.

(a) $z_{0.2}$ and $z_{0.8}$

(b) $t_{0.2}$ and $t_{0.8}$

(c) $\chi_{0.2}$ and $\chi_{0.8}$

Problem 3. Suppose we have a sample of size 16 from a population that is normally distributed and reported to have a mean of 10.

(a) If the true standard deviation σ is 2, then what is the probability that $\bar{x}$ will be greater than 11?

(b) Is the sample standard deviation s is 2 and $\bar{x} = 10.5$, construct a 95% confidence interval around $\bar{x}$.

Problem 4. Suppose that a golfer successfully makes 36 out of 50 of a putts from 10 feet away. Construct a 90% confidence interval for her true probability of making putts from that distance.

Problem 5. Consider the data set

$$x = (-2.1, 1.3, 10.4, 2.4, 5.1, -3.2, 9.9).$$

Assuming that the data is normally distributed, is there sufficient evidence to reject the null hypothesis that the mean is zero using a significance level of $\alpha = 0.1$?

Problem 6. Suppose that a population has a normal distribution with known standard deviation of 3. How large of a sample size do we need if we wish to build a 99% confidence interval so that our error will be less than 0.5?

Problem 7. A team of social scientists have introduced a intervention plan to help families overcome tragic events. For a key outcome, they collected the following data for seven participating families.

Family	**Before**	**After**
1	7	8
2	10	12.5
3	5	5.5
4	14	15
5	20	21.5
6	13	13.5
7	6	7
$\bar{x}$	10.7	11.9
s	5.34	5.55

They computed a pooled sample standard deviation to be $s_p = 5.45$ and used a significance level of $\alpha = 0.05$ to test whether the program has a positive effect.

(a) Complete their pooled variance analysis and show that, using this method, they fail to reject the null hypothesis.

(b) Offer them some advice on how to handle this data set and show that, in fact, there is sufficient evidence to reject the null!

Problem 8. Suppose that, on average, it takes 5.3 days to recover from the common cold. A supplement manufacturer claims that they can reduce this time.

(a) If their trial consists of 12 individuals, propose a one-sided statistical test at a significance level of $\alpha = 0.05$ to assess the manufacturer's claims. Be sure to clearly state the null hypothesis, the alternative hypothesis, the test statistic, and the rejection region.

(b) If the average recovery time among the 12 participants was 4.9 with a sample standard variance of 1.2 days[1], is this sufficient evidence to reject that the hypothesis that the supplement has no effect?

Problem 9. A random sample of 20 students scored an average of 75 on a stat test with a sample variance of $s^2 = 25$. Assuming the scores are normally distributed, construct a two-sided 98% confidence interval for the true population variance σ^2.

[1] This was missing from the original version of the problem.

Problem 10. A testing company believes that well-prepared students will score an average of 75 on a math test they have written. They provide a study guide to a random sample of 25 students who then scored an average of 80 on the test with a sample standard deviation of 4.

Taking $H_0 : \mu = 75$ and $H_1 : \mu > 75$, approximate a p-value for this result.
Hint: Start by converting the result of the study to a t-score (why a t-score and not a z-score?), then assess the probability of a seeing this t-score or higher.

Problem 11. For this problem, assume that the number of errors in a manuscript is Poisson distributed. A typesetter claims that he has an average of 10 typesetting errors per manuscript. We pick a random manuscript produced by this typesetter. We reject the typesetter's claim and will fire him if this randomly selected book has 15 or more errors. In the language of statistical testing, this can be expressed as follows:

$$\begin{aligned} H_0 &: \mu = 10 \\ H_1 &: \mu > 10 \\ \text{Test Statistic: } X &= \text{ number of errors in manuscript} \\ \text{Rejection region: } X &\geq 15. \end{aligned}$$

(a) What is the significance level of this test?

(b) If the true mean is $\mu = 18$, what is the power of this test?

Problem 12. For each of the following, decide whether you should reject, or fail to reject the null hypothesis.

(a) Population proportion.

- Sample size = 100.
- $H_0 : \ p = 0.6.$
- $H_1 : \ p < 0.6.$
- Estimation from population: $\hat{p} = 0.57$.
- Significance level: $\alpha = 0.05$.

(b) Your lab uses a significance level of $\alpha = 0.3$. The reported p-value of a study is 0.035. Do you

REJECT or FAIL TO REJECT?

(c) Your lab only accepts studies that have a probability of false positives less than 0.1. The p-value of a study you read is 0.01. Do you

REJECT or FAIL TO REJECT?